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ON APPLICABILITY OF THE BOUNDARY CONDITION OF THE FIRST KIND IN PROBLEMS OF HEAT TRANSFER IN HEAT-RELEASING GRANULAR BEDS

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The range of applicability of the boundary condition of the first kind at entry into a heat-releasing granular bed has been investigated. It has been shown that in the cases where the heat-transfer process can be described within the framework of a one-temperature (homogeneous) model, it is allowable to use this condition for RePr >> 1. When it is necessary to allow for the phase-temperature difference, the boundary condition of the first kind can be used for the parametric value $\widehat{Q} \le 0.02$. In the remaining regimes, the generalized Danckwerts condition allowing for the preheating of the heat-transfer agent should be used.

Keywords: *Danckwerts condition, boundary condition of the first kind, one-temperature model, two-temperature model, preheating of the heat-transfer agent, heat-releasing granular bed.*

Introduction. Heat-releasing granular beds are a variety of disperse systems (layers of coated fuel particles of the nuclear fuel of atomic power plants, layers of solid-fuel particles in their layer combustion, heat-releasing beds of biological origin, etc.) of practical importance. Heat release in solid particles creates a special character of the temperature field in the system, which is affected by a number of factors: heat-release power, rate of filtration of the heat-transfer agent, particle size, etc. Description of the process is based on the use of a two-temperature model, which is represented in the simplest one-dimensional case by the system of equations

$$c_f J_f \frac{dT_f}{dx} = \frac{d}{dx}\left(\varepsilon \lambda_f \frac{dT_f}{dx}\right) + \frac{6\,(1-\varepsilon)\,\alpha}{d}\,(T_s - T_f)\,, \tag{1}$$

$$0 = \frac{d}{dx}\left((1-\varepsilon)\,\lambda_s \frac{dT_s}{dx}\right) + \frac{6\,(1-\varepsilon)\,\alpha}{d}\,(T_f - T_s) + Q\,(1-\varepsilon)\,. \tag{2}$$

Under the conditions where the phase-temperature difference can be disregarded (e.g., in small heat release), we have $T_f \approx T_s = T$ and model (1)–(2) becomes the one-temperature (quasihomogeneous) model

$$c_f J_f \frac{dT}{dx} = \frac{d}{dx}\left(\lambda \frac{dT}{dx}\right) + Q\,(1-\varepsilon)\,, \tag{3}$$

where $\lambda = \varepsilon\lambda_f + (1-\varepsilon)\lambda_s$ is the effective longitudinal thermal conductivity of the granular bed.

In [1], on the basis of a numerical analysis of (1)–(2), we have obtained an interpolation expression for the bed-average relative difference of the phase temperatures:

$$\eta = \frac{\langle T_s - T_f \rangle}{T_0} = 0.25\tilde{Q}^{0.73}\,. \tag{4}$$

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It is clear that the parameter η characterizes the applicability of the two-dimensional model, whereas the number $\tilde{Q} = Q(1-\varepsilon)/d(c_f J_f T_0)$ makes it possible to determine the ranges of application of different models. If it is assumed that $\eta \approx 0.01$, expression (4) yields that for

$$\tilde{Q} < 0.01 \tag{5}$$

we can use the one-temperature model (3).

Boundary Conditions for the One- and Two-Temperature Models. In [1], we have established the complete system of boundary conditions for system (1)–(2) which reflect the physical conditions of entry and exit of the heat-transfer agent (gas, liquid, or their mixture):

$$x = 0\,, \quad c_f J_f (T_f - T_0) = (1-\varepsilon)\,\lambda_s \frac{dT_s}{dx} + \varepsilon \lambda_f \frac{dT_f}{dx}\,, \tag{6}$$

$$(1-\varepsilon)\,\lambda_s \frac{dT_s}{dx} = \alpha_0 (T_s - T_0) = c_f J_f (T_0' - T_0)\,, \tag{7}$$

$$x = H\,, \quad \frac{dT_f}{dx} = 0\,, \tag{8}$$

$$\frac{dT_s}{dx} = 0\,. \tag{9}$$

Conditions (6), (8) are the generalized Danckwerts conditions; condition (6) includes the influence of the preheating of the heat-transfer agent, which is described by Eq. (7). At $T_f \approx T_s = T$, system (6)–(9) is reduced to the classical Danckwerts conditions [2]:

$$x = 0\,, \quad c_f J_f (T - T_0) = \lambda \frac{dT}{dx}\,, \tag{10}$$

$$x = H\,, \quad \frac{dT}{dx} = 0\,. \tag{11}$$

As is seen, condition (7) becomes superfluous in this case. Physically this means that in formulating the one-temperature model, we need not consider the preheating of the heat-transfer agent (which, certainly, always occurs). Consequently, this model ignores the thermal prehistory of entry of the heat-transfer agent into the bed (its preheating which can be fairly large [3]) in addition to making the thermal pattern in the bed itself obviously rougher (equating the phase temperatures). We note that the preheating of the heat-transfer agent is accompanied by the internal return flow of heat in the system and this has no effect on the total heat balance

$$c_f J_f (T_f(H) - T_0) = Q\,(1-\varepsilon)\,H\,, \tag{12}$$

since the heat flux to the bed, with account for (7), is equal to

$$c_f J_f T_0' - (1-\varepsilon)\,\lambda_s \frac{dT_s}{dx} = c_f J_f T_0\,. \tag{13}$$

Formulation of the correct boundary conditions (6)–(9) and (10), (11) raises the question of whether the use of other, more simple conditions is legitimate. In the literature, e.g., [4–6], the boundary condition of the first kind

$$x = 0\,, \quad T_f = T_0\,; \quad T = T_0 \tag{14}$$

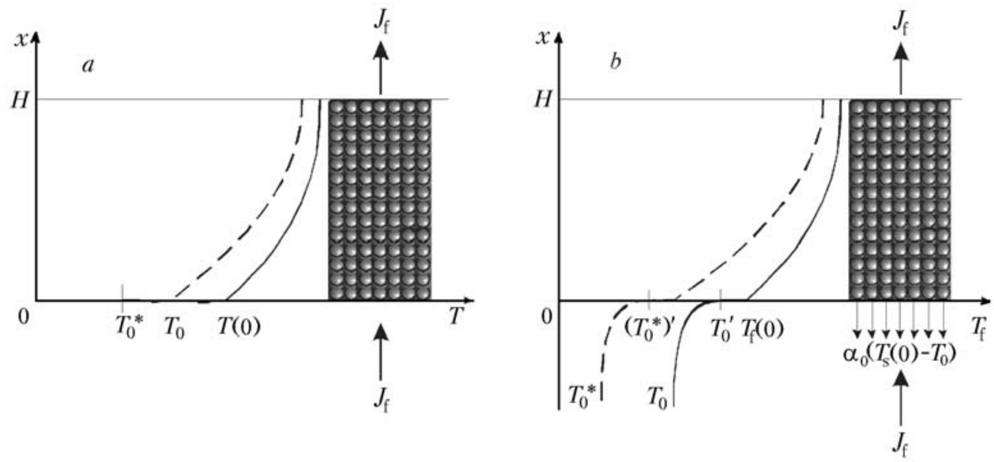

Fig. 1. Temperature distribution of the heat-transfer agent: a) one-temperature model; b) two-temperature model.

is most often used for the two- and one-temperature models respectively. The present work seeks to elucidate the physical conditions of applicability of boundary condition (14) in problems of modeling of heat transfer in heat-releasing granular beds.

Analysis of the Temperature Distribution of the Heat-Transfer Agent at Entry into the Bed and of the Total Heat Balance with the Boundary Condition of the First Kind. *One-Temperature Model.* In the presence of the longitudinal thermal conductivity of the bed, the condition $T(0) = T_0$ automatically brings into existence a certain fictitious inlet temperature of the heat-transfer agent T_0^* which is determined, in accordance with (10), from the equation

$$c_f J_f (T_0 - T_0^*) = \lambda \left. \frac{dT}{dx} \right|_{x=0} . \tag{15}$$

The quantity $\lambda \left. \frac{dT}{dx} \right|_{x=0}$ represents the heat flux going to heat the heat-transfer agent at entry into the bed from T_0^* to T_0. Since the quantity $\lambda \left. \frac{dT}{dx} \right|_{x=0}$ is real, and heating from T_0^* to T_0 is conventional in essence, we arrive at the conclusion that the heat flux $\lambda \left. \frac{dT}{dx} \right|_{x=0}$ goes from the bed to "nowhere." This circumstance is pronounced in analyzing the total heat balance

$$c_f J_f (T(H) - T_0^*) = Q (1 - \varepsilon) H . \tag{16}$$

With account for (15), we have

$$c_f J_f (T(H) - T_0) = Q (1 - \varepsilon) H - \lambda \left. \frac{dT}{dx} \right|_{x=0} . \tag{17}$$

A comparison of (17) to the real heat-balance equation (12) shows that the use of the boundary condition of the first kind leads to an upset total heat balance in the system. Clearly, this is due to the neglect of the heat flux which goes to heat the heat-transfer agent from T_0 to $T(0)$ at entry into the bed. This circumstance was first noted in [7] for an elliptic heat-conduction equation. The temperature distribution of the heat-transfer agent is diagrammatically shown in Fig. 1a.

Two-Temperature Model. In this case, too, there appears the fictitious inlet temperature, which is determined by the equation following from (6):

$$c_f J_f (T_0 - T_0^*) = (1 - \varepsilon)\, \lambda_s \frac{dT_s}{dx}\bigg|_{x=0} + \varepsilon \lambda_f \frac{dT_f}{dx}\bigg|_{x=0} . \tag{18}$$

The first term of the right-hand side of (18) is the heat flux going to preheat the heat-transfer agent from T_0^* to $(T_0^*)'$, whereas the second term is the heat flux going into heating from $(T_0^*)'$ to T_0 (Fig. 1b). Substituting the quantity T_0^* from (18) into the heat-balance equation

$$c_f J_f (T_f (H) - T_0^*) = Q (1 - \varepsilon) H , \tag{19}$$

we obtain

$$c_f J_f (T_f (H) - T_0) = Q (1 - \varepsilon) H - (1 - \varepsilon)\, \lambda_s \frac{dT_s}{dx}\bigg|_{x=0} - \varepsilon \lambda_f \frac{dT_f}{dx}\bigg|_{x=0} . \tag{20}$$

A comparison of (20) with the real heat-balance equation (12) shows that in this case, too, we have the upset total heat balance. The reason is the neglect of the heat flux going to preheat the heat-transfer agent from T_0 to T_0' and of the flux that increases the temperature from T_0' to $T_f(0)$, as the heat-transfer agent enters the granular bed (Fig. 1b).

As is seen, the condition of the first kind for $x = 0$ does not reflect the actual features of the heat-transfer agent entering the bed and leads to violation of the law of conservation of thermal energy in the system. Consequently, this condition can be used only as a certain approximation that can be justified only under certain conditions. Clearly, to specifically elucidate them we need investigate only the dependence of $\theta_f(0)$ of $\theta(0)$ on the governing factors.

One-Temperature Model with Constant-Power Heat Release. On condition that λ = const, the thermal problem is formulated as follows:

$$\frac{d\theta}{d\xi} = \frac{1}{\mathrm{Pe}} \frac{d^2\theta}{d\xi^2} + \hat{Q} , \tag{21}$$

$$\xi = 0 , \quad \theta = \frac{1}{\mathrm{Pe}} \frac{d\theta}{d\xi} , \tag{22}$$

$$\xi = 1 , \quad \frac{d\theta}{d\xi} = 0 . \tag{23}$$

The solution of (21)–(23) has the form

$$\theta (\xi) = \hat{Q} \left(\frac{1}{\mathrm{Pe}} + \xi - \frac{\exp (\mathrm{Pe} (\xi - 1))}{\mathrm{Pe}} \right). \tag{24}$$

For the sought $\theta(0)$ we obtain

$$\theta (0) = \frac{\hat{Q}}{\mathrm{Pe}} (1 - \exp (- \mathrm{Pe})) . \tag{25}$$

Let us consider two limiting cases:

(a) RePr << 1; here, we have $\lambda/\lambda_f^0 = 1 + 0.5$ RePr ≈ 1 [8] and from (25) we obtain

$$\theta(0) \approx \tilde{Q}\,(\mathrm{Re\,Pr})^{-1}\,; \tag{26}$$

(b) RePr >> 1; in this case we have $\lambda/\lambda_f^0 = 1 + 0.5$ RePr ≈ 0.5 RePr and (25) yields

$$\theta(0) = \frac{Q(1-\varepsilon)\,d}{2c_f J_f T_0}\left(1 - \exp\left(-\frac{2H}{d}\right)\right) \approx \frac{\tilde{Q}}{2}\,. \tag{26a}$$

Thus, the sought relative temperature difference is determined by the numbers Re, Pr, and $\tilde{Q} = Q(1-\varepsilon)d/(c_f J_f T_0)$. Assigning $\dfrac{T(0)-T_0}{T_0} \le 00.1$, we obtain the estimate of the possibility of using the boundary condition of the first kind:

$$\tilde{Q} \le \begin{cases} 0.01\mathrm{Re\,Pr}\,, & \mathrm{Re\,Pr} << 1\,; \\ 0.02\,, & \mathrm{Re\,Pr} >> 1\,. \end{cases} \tag{27}$$

The first inequality in (27) corresponds to a virtually isothermal layer (since $Q \to 0$) and is of no practical interest. A comparison of (27) and (5) enables us to infer that under the conditions of applicability of the one-temperature model, it is allowable to use the boundary condition of the first kind for RePr >> 1. In the remaining cases the error from the use of the boundary condition of the first kind is calculated from (25).

One-Temperature Model with Variable-Power Heat Release. A granular-bed heat exchanger is the characteristic example of practical importance of such a system. In a one-dimensional approximation, the thermal problem is formulated as follows:

$$\frac{d\theta_*}{d\xi} = \frac{1}{\mathrm{Pe}}\frac{d^2\theta_*}{d\xi^2} - 2\mathrm{St_w}\frac{H}{R}\theta_*\,, \tag{28}$$

$$\xi = 0\,, \quad \theta_* - 1 = \frac{1}{\mathrm{Pe}}\frac{d\theta_*}{d\xi}\,, \tag{29}$$

$$\xi = 1\,, \quad \frac{d\theta_*}{d\xi} = 0\,. \tag{30}$$

The solution of (28)–(30) for $\dfrac{T_f(0)-T_0}{T_0}$ has the form

$$\frac{T_f(0)-T_0}{T_0} = \frac{T_w - T_0}{T_0}\,\frac{1}{1 + \dfrac{K_1 - K_2\exp\left(-2\sqrt{\dfrac{\mathrm{Pe}^2}{4} + 2\mathrm{St_w\,Pe}\dfrac{H}{R}}\right)}{2\mathrm{St_w\,Pe}\dfrac{H}{R}\left(1 - \exp\left(-2\sqrt{\dfrac{\mathrm{Pe}^2}{4} + 2\mathrm{St_w\,Pe}\dfrac{H}{R}}\right)\right)}}\,. \tag{31}$$

Let us consider expression (31) for large H and RePr. In this case passage to the limit $H \to \infty$ in (31) at $T_w < T_{cr}$ yields

$$\frac{T_f(0)-T_0}{T_0} = -\frac{T_w - T_0}{T_0}\,\frac{1 - \sqrt{1 + 4\mathrm{St_w}\dfrac{d}{R}}}{1 + \sqrt{1 + 4\mathrm{St_w}\dfrac{d}{R}}}\,. \tag{32}$$

When $\mathrm{St_w}d/R$ are small expression (32) is simplified and, for $\frac{T_f(0) - T_0}{T_0}$, we have

$$\frac{T_f(0) - T_0}{T_0} = \left(\frac{T_w}{T_0} - 1\right) \mathrm{St_w} \frac{d}{R} . \tag{33}$$

Determining $Q_w(1 - \varepsilon)$ as $\frac{2(T_w - T_0)K_w}{R}$, we obtain, from (33), the condition of applicability of the boundary condition of the first kind, which is analogous to (27):

$$\tilde{Q}_w = \frac{Q_w (1 - \varepsilon) d}{c_f J_f T_0} \le 0.02 . \tag{34}$$

Two-Temperature Model. With account for the equation of motion [1] and the equation of state for a gaseous heat-transfer agent, the system of equations (1), (2), and (6)–(9) in dimensionless form will be as follows:

$$\frac{d\theta_f}{d\xi} = \frac{d}{d\xi}\left(\frac{1}{\mathrm{Pe_f}} \frac{d\theta_f}{d\xi}\right) + \frac{6 (1 - \varepsilon) H}{d} \mathrm{St} (\theta_s - \theta_f) , \tag{35}$$

$$0 = \frac{d}{d\xi}\left(\frac{1}{\mathrm{Pe_s}} \frac{d\theta_s}{d\xi}\right) + \frac{6 (1 - \varepsilon) H}{d} \mathrm{St} (\theta_f - \theta_s) + \hat{Q} , \tag{36}$$

$$\bar{J}_f \rho'_f \frac{d}{d\xi}\left(\frac{1}{\rho'_f}\right) = - D\frac{dp'}{d\xi} - 150\frac{(1 - \varepsilon)^2}{\varepsilon^3} \mathrm{Re} - 1.75 \frac{1 - \varepsilon}{\varepsilon^3} \mathrm{Re}^2 , \tag{37}$$

$$\rho'_f = \frac{p'}{\theta + 1} , \tag{38}$$

$$\xi = 0 , \quad \theta_f = \frac{1}{\mathrm{Pe_f}} \frac{d\theta_f}{d\xi} + \frac{1}{\mathrm{Pe_s}} \frac{d\theta_s}{d\xi} , \quad \frac{1}{\mathrm{Pe_s}} \frac{d\theta_s}{d\xi} = \mathrm{St_0}\theta_s , \ \xi = 1, p' = 1, \ \frac{d\theta_f}{d\xi} = \frac{d\theta_s}{d\xi} = 0 . \tag{39}$$

In setting the boundary condition of the first kind, we take

$$\xi = 0 , \quad \theta_f = 0 \tag{40}$$

instead of (37). As the model's parameters we use the dependences [1, 9–11]

$$\alpha = \begin{cases} 1.6 \cdot 10^{-2} \frac{\lambda_f^0}{d} \left(\frac{\mathrm{Re}}{\varepsilon}\right)^{1/3} \mathrm{Pr}^{1/3} , & \frac{\mathrm{Re}}{\varepsilon} \le 200 ; \\ 0.4 \frac{\lambda_f^0}{d} \left(\frac{\mathrm{Re}}{\varepsilon}\right)^{2/3} \mathrm{Pr}^{1/3} , & \frac{\mathrm{Re}}{\varepsilon} > 200 , \end{cases} \tag{41}$$

$$\lambda_f = \lambda_f^0 \left(1 + 0.5 \frac{\mathrm{Re}}{\varepsilon} \mathrm{Pr}\right), \tag{42}$$

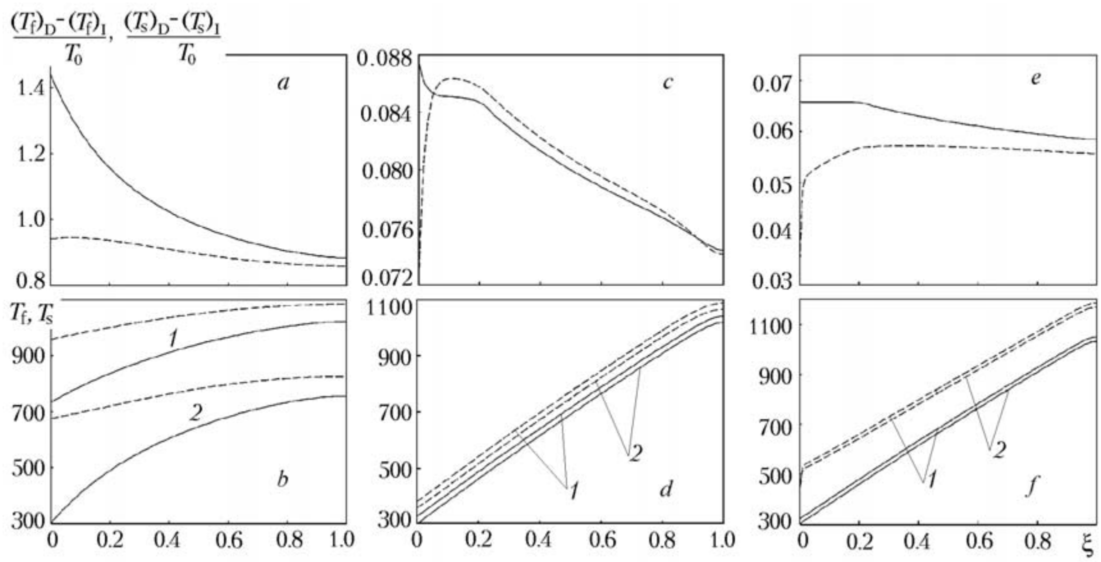

Fig. 2. Profiles of temperatures and dimensionless temperature differences of the heat-transfer agent (Pr = 0.72) and particles, which are calculated with the boundary Danckwerts condition and the boundary condition of the first kind at entry into the granular bed $\tilde{Q}$ = 0.13 and d = 0.002 m (solid curves, heat-transfer agent, dashed curves, particles): 1) boundary Danckwerts condition; 2) boundary condition of the first kind; a and b) Re = 0.57 and $Q(1-\varepsilon) = 10^5$ W/m^3, c and d) 57 and $Q(1-\varepsilon) = 10^7$ W/m^3, e and f) 5764 and $Q(1-\varepsilon) = 10^9$ W/m^3. T_f and T_s, K.

$$\lambda_s = \frac{\lambda_c - \varepsilon\lambda_f}{1-\varepsilon} + \lambda_r\,, \tag{43}$$

$$\lambda_c = \lambda_f^0 \left(\frac{\lambda_s^0}{\lambda_f^0}\right)^{(1-\varepsilon)\left(\frac{\lambda_s^0}{\lambda_f^0}\right)^{-0.06}}\,, \tag{44}$$

$$\lambda_r = \frac{0.3024}{\kappa+\sigma}\left(\frac{T_s}{100}\right)^3\,, \tag{45}$$

$$\alpha_0 = 0.5 c_f J_f \mathrm{Re}^{-0.5}\mathrm{Pr}^{-0.6}\,; \tag{46}$$

air:

$$\rho_f = 0.00352 p/T_f,\ \lambda_f^0 = 0.00021 T_f^{0.84},\ \mu_f = 2.64\cdot 10^{-7} T_f^{0.74} (\mathrm{Pr} = 0.72);$$

the liquid-metal heat-transfer agent:

$$\rho_f = 970\ \mathrm{kg/m^3}\,,\quad \lambda_f^0 = 134\ \mathrm{W/(m{\cdot}K)}\,,\quad \mu_f = 0.00055\ \mathrm{kg/(m{\cdot}sec)}\quad (\mathrm{Pr} = 0.005)\,. \tag{47}$$

Figure 2 shows results of calculation of the T_s and T_f fields for different heat-release powers in the case of air cooling. The same figure gives the relative differences of the particle and heat-transfer-agent temperatures calculated with the Danckwerts condition and the boundary condition of the first kind. To calculate the quantity $\theta_f(0)$ we have obtained the interpolation formula ($Q(1-\varepsilon) = 10^5$–10^9 W/m^3):

$$\theta_f(0) = \begin{cases} 6.5\tilde{Q}\,\mathrm{Re}^{-0.5}\mathrm{Pr}^{-0.26}, & \left[\begin{array}{l}\mathrm{Re} \le 200, \quad \mathrm{Pr} = 0.72;\\ \mathrm{Re} \le 2000, \quad \mathrm{Pr} = 0.005;\end{array}\right. \\ 0.5\tilde{Q}, & \left[\begin{array}{l}\mathrm{Re} > 200, \quad \mathrm{Pr} = 0.72;\\ \mathrm{Re} > 2000, \quad \mathrm{Pr} = 0.005.\end{array}\right. \end{cases} \tag{48}$$

The second formula in (48) coincides with (26a), since at elevated Re values, the two-temperature model is close to the one-temperature model because of the large interphase exchange. It is noteworthy that the minimum values of J_f in the calculations have been determined from the formula

$$(J_f)_{\min} = \frac{Q\,(1-\varepsilon)\,H}{(T_{cr} - T_0)\,c_f}, \tag{49}$$

which is a consequence of heat-balance condition (12) at $T_f(H) = T_{cr}$. The conditions of applicability of the boundary condition of the first kind $\theta_f(0) \le 0.01$ in accordance with (48) are as follows

$$\tilde{Q} = \begin{cases} 1.5\cdot 10^{-3}\,\mathrm{Re}^{0.5}\mathrm{Pr}^{0.26}, & \left[\begin{array}{l}\mathrm{Re} \le 200, \quad \mathrm{Pr} = 0.72;\\ \mathrm{Re} \le 2000, \quad \mathrm{Pr} = 0.005;\end{array}\right. \\ 0.02, & \left[\begin{array}{l}\mathrm{Re} > 200, \quad \mathrm{Pr} = 0.72;\\ \mathrm{Re} > 2000, \quad \mathrm{Pr} = 0.005.\end{array}\right. \end{cases} \tag{50}$$

With account for (5), formulas (50) are applicable for $\tilde{Q} > 0.01$. For $\tilde{Q}$ values higher than those determined by (50), we must use the generalized Danckwerts condition (6). The error of setting the boundary condition of the first kind is evaluated according to (48).

Conclusions. The two-temperature model describing the thermal situation in the bed in detail requires that the correct boundary condition, i.e., the generalized Danckwerts condition (6), which also reflects the prehistory of entry of the heat-transfer agent into the granular bed, be set. Setting of the boundary condition of the first kind is allowable only for $\hat{Q} \le 0.2$. When the process of heat transfer can be described within the framework of the one-temperature model, it is allowable to use this condition for RePr >> 1.

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NOTATION

c_f, specific heat of the heat-transfer agent at constant pressure, J/(kg·K); d, particle diameter, m; $D = p_{atm}d^3\rho_f/(H\mu_f^2)$; H, height of the granular bed, m; $J_f = \rho_f u$, mass flow rate of the heat-transfer agent, kg/(m^2·sec); $\bar{J}_f = J_f^2 d^3/(\varepsilon^2 H m_f^2)$; $K_1 = \frac{\mathrm{Pe}}{2} + \sqrt{\frac{\mathrm{Pe}^2}{4} + 2\mathrm{St_w Pe}\frac{H}{R}}$; $K_2 = \frac{\mathrm{Pe}}{2} - \sqrt{\frac{\mathrm{Pe}^2}{4} + 2\mathrm{St_w Pe}\frac{H}{R}}$; K_w, heat-transfer coefficient, W/(m^2·K); $\mathrm{Pe} = \frac{c_f J_f H}{\lambda}$, $\mathrm{Pe}_f = \frac{c_f J_f H}{\varepsilon\lambda_f}$, and $\mathrm{Pe}_s = \frac{c_f J_f H}{(1-\varepsilon)\lambda_s}$, Péclet numbers; $\mathrm{Pr} = \frac{c_f\mu_f}{\lambda_f^0}$, Prandtl number; p, pressure, Pa; $p^1 = p/p_{atm}$; Q, heat-release power, W/m^3; $\hat{Q} = \frac{Q(1-\varepsilon)H}{c_f J_f T_0}$, $\tilde{Q} = \frac{Q(1-\varepsilon)d}{c_f J_f T_0}$; $\mathrm{Re} = \frac{J_f d}{\mu_f}$, Reynolds number; $\mathrm{St} = \frac{\alpha}{c_f J_f}$; $\mathrm{St_w} = \frac{K_w}{c_f J_f}$; $\mathrm{St}_0 = \frac{\alpha_0}{c_f J_f}$, Stanton numbers; T, temperature, K; T_0, inlet temperature of the heat-transfer agent, K; T_w, ambient temperature, K; T_0', temperature of the heat-transfer agent for $x \to -0$, K; T_0^*, fictitious inlet temperature of the heat-transfer agent, K; $(T_0^*)'$, fictitious temperature of the heat-transfer agent for $x \to -0$, K; u, filtration rate of the heat-transfer agent, m/sec; x, longitudinal coordinate, m; α, coefficient of interphase heat exchange, W/(m^2·K); α_0, coefficient of heat exchange of the heat-transfer agent with the particle matrix, W/(m^2·K); ε, porosity; $\theta = (T - T_0)/T_0$; $\theta_* = \dfrac{T - T_w}{T_0 - T_w}$; κ, absorption coefficient of the disperse medium, 1/m; λ, longitudinal thermal conductivity of the bed, W/(m·K); λ_f and λ_s, longitudinal thermal conductivities of the heat-transfer agent and the particle matrix, W/(m·K); λ_f^0, molecular thermal conductivity of the heat-transfer agent, W/(m·K); λ_s^0, thermal conductivity of the particle material, W/(m·K); μ_f, dynamic viscosity of the heat-transfer agent, kg/(m·sec); $\xi = x/H$; ρ_f, density of the heat-transfer agent, kg/m^3; σ, scattering coefficient of the disperse medium, 1/m. Subscripts: f, heat-transfer agent; cr, critical; 0, at entry, inlet; s, particles; w, apparatus wall; c, conductive; r, radiative; min, minimum.

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